

Proton Stability in $S_4 \times Z_2$ Flavor Symmetric Extra U(1) Model

Yasuhiro Daikoku^{a1} and Hiroshi Okada^{b2}

^a *Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

^b *School of Physics, KIAS, Seoul 130-722, Korea*

Abstract

We consider proton stability based on E_6 inspired extra U(1) model with $S_4 \times Z_2$ flavor symmetry. In this model, a long life time of proton is realized by the flavor symmetry in several ways. We classify the suppression mechanisms of proton-decay and explain how the flavor symmetry works. There is an interesting solution, such as, in a special direction of vacuum expectation value (VEV), baryon number violating interactions are canceled. In the case that the suppression of proton decay is realized by the appropriate size of VEV, the allowed region of VEV exists when the exotic quark mass is $O(\text{TeV})$. From the constraint for the life time of exotic quark, the right handed neutrino mass should be in narrow range around 10^{12}GeV .

¹yasu_daikoku@yahoo.co.jp

²hokada@kias.re.kr

1 Introduction

Supersymmetry is an elegant solution of hierarchy problem of Standard Model (SM) [1] and gives a new view point of generation structure of leptons and quarks. As a simple supersymmetric extension of SM suffers from non-conservation of baryon number, we must introduce R-parity symmetry in order to avoid too rapid proton decay. This means we can not construct any consistent superpotential based on only SM gauge symmetry. Even if we introduce R-parity, the superpotential of minimal supersymmetric standard model (MSSM) is not perfect one. The superpotential of MSSM suffers from μ -problem such as we must tune the scale of μ -parameter to be $O(\text{TeV})$, which is much smaller than Planck scale. Therefore the R-parity symmetry should be replaced by other symmetry. The information what symmetry should be introduced may be extracted from the structure of Yukawa interactions because these interactions are derived from superpotential.

The appropriate start point is given by introducing an additional $U(1)$ gauge symmetry to forbid μ -term [2]. In this frame work, several new superfields such as singlet S , exotic quarks G, G^c , must be introduced to cancel gauge anomaly. Then the baryon number violating interactions in superpotential are replaced by single exotic quark interactions, which make it easy to suppress proton decay by the new symmetry.

Considering the Yukawa interactions, we can guess about which symmetry we should introduce. Strangely, the mixing angle θ_{23} of Maki-Nakagawa-Sakata (MNS) matrix is almost maximal. Many authors discussed non-Abelian discrete flavor symmetries to understand the structure of the MNS matrix. The flavor symmetry may be a good candidate for replacing R-parity symmetry. At previous work, we explained $S_4 \times Z_2$ flavor symmetry not only realizes maximal mixing angle θ_{23} but also suppresses proton decay based on $SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_X \times U(1)_Z$ gauge symmetry [3]. Therefore there is a deep connection between the proton stability and the existence of generation in supersymmetric model.

In this paper we give more detailed estimation of proton life time and classify the mechanisms to suppress proton decay. At first we give definition of our model in section 2, and give classification of suppression mechanisms of proton decay in section 3. In section 4, we check that our superfield assignment realizes neutrino mass square differences and MNS matrix including new experimental value of θ_{13} . We explain the origin of $U(1)_Z$ breaking scale in section 5. Finally we give conclusion of our analysis in section 6.

2 $S_4 \times Z_2$ flavor symmetric extra $U(1)$ model

At first we explain the basic structure of our model. We extend the gauge symmetry to $SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_X \times U(1)_Z$ which is the maximal subgroup of E_6 . In order to cancel gauge anomaly, we must add new superfields, such as SM singlet S , exotic quark G, G^c (hereafter we call them g-quark) and right handed neutrino (RHN) N^c . We can embed these superfields with MSSM superfields $Q, U^c, D^c, L, E^c, H^U, H^D$ into **27** of E_6 [4]. As the singlet S develops VEV and breaks $U(1)_X$ gauge symmetry, $O(\text{TeV})$ scale μ -term is induced naturally. In order to break $U(1)_Z$ and generate a large Majorana mass of RHN, we add SM singlet Φ, Φ^c . The gauge representations of superfields are given in Table 1 [3].

	Q	U^c	E^c	D^c	L	N^c	H^D	G^c	H^U	G	S	Φ	Φ^c
$SU(3)_c$	3	3^*	1	3^*	1	1	1	3^*	1	3	1	1	1
$SU(2)_W$	2	1	1	1	2	1	2	1	2	1	1	1	1
$y = 6Y$	1	-4	6	2	-3	0	-3	2	3	-2	0	0	0
x	1	1	1	2	2	0	-3	-3	-2	-2	5	0	0
z	-1	-1	-1	2	2	-4	-1	-1	2	2	-1	8	-8
R	-	-	-	-	-	-	+	+	+	+	+	+	+

Table 1: G_2 assignment of superfields. Where x, y and z are charges of $U(1)_X, U(1)_Y$ and $U(1)_Z$, respectively. Y is hypercharge. After the gauge symmetry breaking of three $U(1)$ s, R-parity symmetry $R = \exp[i\pi(3x - 8y + 15z)/20]$ is unbroken.

Under the gauge symmetry given in Table 1, the renormalizable superpotential is given by

$$\begin{aligned}
W = & Y^U H^U Q U^c + Y^D Q D^c H^D + Y^E H^D L E^c + Y^N H^U L N^c + Y^M \Phi N^c N^c + \lambda S H^U H^D + k S G G^c \\
& + M \Phi \Phi^c + Y^{QQ} G Q Q + Y^{UD} G^c U^c D^c + Y^{UE} G E^c U^c + Y^{LQ} G^c L Q + Y^{ND} G N^c D^c.
\end{aligned} \tag{1}$$

In this superpotential, unwanted terms are included in the second line. The first term of the second line is the mass term of singlets Φ, Φ^c which prevent singlets from developing VEVs. The other five terms of the second line are single g-quark interactions, which break baryon and lepton number and induce rapid proton decay. In the first line, we must take care of the flavor changing neutral currents (FCNCs) induced by extra Higgs bosons [5]. Therefore the superpotential Eq.(1) is not consistent at the present stage.

In order to stabilize proton, we introduce $S_4 \times Z_2$ flavor symmetry. If we assign G, G^c to S_4 triplet and quarks and leptons to doublet or singlet, the single g-quark interaction is forbidden. However, as the g-quark must never be stable from phenomenological reason, we assign Φ^c to S_4 triplet to break the flavor symmetry slightly. In order to realize the maximal mixing angle of θ_{23} in the MNS matrix and suppress the Higgs-mediated FCNCs, we assign the superfields in our model as given in Table 2 [6].

In the non-renormalizable part of superpotential, the single g-quark interactions which contribute to the g-quark decay are given as follows

$$W_B = \frac{y^{QQ}}{M_P^2} \Phi \Phi^c Q Q G + \frac{y^{UD}}{M_P^2} \Phi \Phi^c G^c U^c D^c + \frac{y^{EU}}{M_P^2} \Phi \Phi^c G E^c U^c + \frac{y^{QL}}{M_P^2} \Phi \Phi^c G^c L Q. \quad (2)$$

The detail of W_B depends on the Z_2 charge assignment of p_q and p_g .

	Q_1	Q_2	Q_3	U_1^c	U_2^c	U_3^c	D_1^c	D_2^c	D_3^c
S_4	1	1	1	1	1	1	1	1	1
Z_2	p_q	p_q	p_q	p_q	p_q	p_q	p_q	p_q	p_q
	E_1^c	E_2^c	E_3^c	L_i	L_3	N_i^c	N_3^c	H_i^D	H_3^D
S_4	1	1	1'	2	1	2	1	2	1
Z_2	+	-	+	-	-	+	-	-	+
	H_i^U	H_3^U	S_i	S_3	G_a	G_a^c	Φ_i	Φ_3	Φ_a^c
S_4	2	1	2	1	3	3	2	1	3
Z_2	-	+	-	+	p_g	p_g	+	+	+

Table 2: $S_4 \times Z_2$ assignment of superfields (Where the index i of the S_4 doublets runs $i = 1, 2$, and the index a of the S_4 triplets runs $a = 1, 2, 3$. The charges of quarks and g-quarks; p_q and p_g , take \pm .)

3 Classification of the suppression mechanism of proton decay

Depending on the p_q and p_g , the flavor symmetry works in different ways to suppress proton decay. In this section we classify the suppression mechanism and estimate the allowed parameter range.

(1) Leptoquark solution

If we assign $(p_q, p_g) = (+, -)$, it results $y^{QQ} = y^{UD} = 0$ in Eq.(2). In this case we can assign lepton number (L) and baryon number (B) of G to $(L, B) = (+1, 1/3)$ and those of G^c to $(L, B) = (-1, -1/3)$, then the baryon number is conserved. Therefore proton becomes stable and VEVs of Φ, Φ^c are not bounded from above. In this solution, G, G^c are well known as leptoquarks. Note that the VEVs of Φ, Φ^c are bounded from below, because the life time of g-quark must be shorter than 0.1 sec, otherwise the success of BBN is spoiled [7].

If we assign $(p_q, p_g) = (-, -)$, then it results $y^{QQ} = y^{UD} = y^{QL} = 0$ and g-quark becomes leptoquark which couples only to right handed charged leptons E_1^c, E_3^c . Therefore the decay mode of our leptoquark depends on the flavor charge assignment.

(2) Cancellation solution

If we assign $(p_q, p_g) = (+, +)$, it results $y^{QL} = 0$. In this case, as we cannot define the lepton number and baryon number of g-quark, these numbers are not conserved. Here we investigate proton decay interactions which are induced by scalar g-quarks exchange. In the present flavor assignment, the superpotential which contributes to the proton decay is given by

$$W_B = \frac{y_a}{M_P^2} U_a^c E_3^c [\sqrt{3}(G_2 \Phi_2^c - G_3 \Phi_3^c) \Phi_2 - (G_2 \Phi_2^c + G_3 \Phi_3^c - 2G_1 \Phi_1^c) \Phi_1] \\ + \frac{y_{ab}}{M_P^2} Q_a Q_b \Phi_3 (\Phi_1^c G_1 + \Phi_2^c G_2 + \Phi_3^c G_3) + \frac{y'_{ab}}{M_P^2} U_a^c D_b^c \Phi_3 (\Phi_1^c G_1^c + \Phi_2^c G_2^c + \Phi_3^c G_3^c)$$

$$\begin{aligned}
& + \frac{z_{ab}}{M_P^2} Q_a Q_b [\sqrt{3}(G_2 \Phi_2^c - G_3 \Phi_3^c) \Phi_1 + (G_2 \Phi_2^c + G_3 \Phi_3^c - 2G_1 \Phi_1^c) \Phi_2] \\
& + \frac{z'_{ab}}{M_P^2} U_a^c D_b^c [\sqrt{3}(G_2^c \Phi_2^c - G_3^c \Phi_3^c) \Phi_1 + (G_2^c \Phi_2^c + G_3^c \Phi_3^c - 2G_1^c \Phi_1^c) \Phi_2].
\end{aligned} \tag{3}$$

Note that the contribution from E_1^c is omitted because it is shown to be τ lepton in section 4 and does not contribute to proton decay. Integrating out scalar g-quarks, we get the effective four-Fermi interactions as follows

$$\begin{aligned}
\mathcal{L} &= \frac{C_{GG}}{M_P^4 M_G^2} \sum_{abc} \lambda_{abc} \mu^c u_a^c \bar{q}_b \bar{q}_c + \frac{C_{GG^c}}{M_P^4 M_G^2} \sum_{abc} \lambda'_{abc} \mu^c u_a^c u_b^c d_c^c, \\
\lambda_{abc} &= y_a y_{bc} \left[\sqrt{3} \langle \Phi_3 \rangle \langle \Phi_2 \rangle \left(\langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) + \langle \Phi_3 \rangle \langle \Phi_1 \rangle \left(2 \langle \Phi_1^c \rangle^2 - \langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) \right] \\
&+ y_a z_{bc} \left[2 \langle \Phi_1 \rangle \langle \Phi_2 \rangle \left(\langle \Phi_2^c \rangle^2 + \langle \Phi_3^c \rangle^2 - 2 \langle \Phi_1^c \rangle^2 \right) - \sqrt{3} \left(\langle \Phi_1 \rangle^2 - \langle \Phi_2 \rangle^2 \right) \left(\langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) \right], \\
\lambda'_{abc} &= y_a y'_{bc} \left[\sqrt{3} \langle \Phi_3 \rangle \langle \Phi_2 \rangle \left(\langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) + \langle \Phi_3 \rangle \langle \Phi_1 \rangle \left(2 \langle \Phi_1^c \rangle^2 - \langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) \right] \\
&+ y_a z'_{bc} \left[2 \langle \Phi_1 \rangle \langle \Phi_2 \rangle \left(\langle \Phi_2^c \rangle^2 + \langle \Phi_3^c \rangle^2 - 2 \langle \Phi_1^c \rangle^2 \right) - \sqrt{3} \left(\langle \Phi_1 \rangle^2 - \langle \Phi_2 \rangle^2 \right) \left(\langle \Phi_2^c \rangle^2 - \langle \Phi_3^c \rangle^2 \right) \right], \tag{4}
\end{aligned}$$

where $\mu^c = e_3^c$ and mean scalar g-quark mass M_G and dimensionless coefficients C_{GG}, C_{GG^c} are defined in appendix. Note that the masses of scalar g-quarks are degenerated due to the S_4 symmetry. Interestingly, it results $\lambda_{abc} = \lambda'_{abc} = 0$ in the special VEV direction such as

$$\langle \Phi_1^c \rangle = \langle \Phi_2^c \rangle = \langle \Phi_3^c \rangle. \tag{5}$$

In this case proton decay is forbidden. This means the contributions from three scalar g-quarks are canceled.

(3) Suppression solution

In the case that there is no remarkable cancellation, the size of $\langle \Phi \rangle$ must be in appropriate region where the constraints for proton and g-quark life time are satisfied at the same time [8]. In order to suppress proton decay, the VEV of Φ must not be too large. At first we estimate the upper bound of the VEV. As there are many unknown parameters in W_B and the VEV direction of Φ, Φ^c is also unknown, we make several assumption for simplicity. At first, we assume there is no mixing between scalar g-quarks G and G^c and put $C_{GG} = 1, C_{GG^c} = 0$. Next, we change the assignment of Q, U^c, D^c and G, G^c to $(S_4, Z_2) = (\mathbf{1}', +)$ and $(S_4, Z_2) = (\mathbf{3}, +)$ respectively in Table 2 and replace the superpotential in Eq.(3) by

$$\begin{aligned}
W_B &= \frac{y_a}{M_P^2} U_a^c E_3^c [\sqrt{3}(G_2 \Phi_2^c - G_3 \Phi_3^c) \Phi_1 + (G_2 \Phi_2^c + G_3 \Phi_3^c - 2G_1 \Phi_1^c) \Phi_2] \\
&+ \frac{z_a}{M_P^2} U_a^c E_3^c \Phi_3 (G_1 \Phi_1^c + G_2 \Phi_2^c + G_3 \Phi_3^c) \\
&+ \frac{y_{ab}}{M_P^2} Q_a Q_b \Phi_3 (G_1 \Phi_1^c + G_2 \Phi_2^c + G_3 \Phi_3^c) \\
&+ \frac{z_{ab}}{M_P^2} Q_a Q_b [\sqrt{3}(G_2 \Phi_2^c - G_3 \Phi_3^c) \Phi_1 + (G_2 \Phi_2^c + G_3 \Phi_3^c - 2G_1 \Phi_1^c) \Phi_2],
\end{aligned} \tag{6}$$

where we assume $y_a = z_{ab} = 0$ and the contribution from y^{UD} is omitted. Finally we tune the VEV direction as follows

$$\langle \Phi_1^c \rangle = \langle \Phi_2^c \rangle = \langle \Phi_3^c \rangle = \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \langle \Phi_3 \rangle = \frac{V}{\sqrt{3}}. \tag{7}$$

Including the renormalization factor A_{RF} , the effective four-Fermi interactions at 1GeV is given by [9]

$$\mathcal{L} = \sum_{abc} \frac{A_{RF} z_a y_{bc} V^4}{3 M_P^4 M_G^2} \mu^c u_a^c \bar{q}_b \bar{q}_c, \tag{8}$$

$$A_{RF} = (A_{RF}^y)_S (A_{RF}^z)_S (A_{RF})_L, \tag{9}$$

where we estimate the short distance part of A_{RF} by the 1-loop renormalization group equations as follows

$$(4\pi) \frac{d \ln z_a}{d \ln \mu} = -\frac{16}{3} \alpha_s, \tag{10}$$

$$(4\pi) \frac{d \ln y_{ab}}{d \ln \mu} = -\frac{24}{3} \alpha_s. \tag{11}$$

Here only QCD correction is accounted. This approximation is not bad because the beta function of the coupling constant of strong interaction g_s vanishes at 1-loop level in our model, which makes the contribution of α_s dominant in the RGEs of z_a and y_{ab} . Solving Eq.(10) and Eq.(11), if we put $\alpha_s(M_Z) = 0.118$, we get

$$(A_{RF}^z)_S = \left(\frac{M_P}{M_Z}\right)^{4\alpha_s/3\pi} = \left(\frac{2.43 \times 10^{18}}{91}\right)^{0.05008} = 6.647, \quad (12)$$

$$(A_{RF}^y)_S = \left(\frac{M_P}{M_Z}\right)^{2\alpha_s/\pi} = \left(\frac{2.43 \times 10^{18}}{91}\right)^{0.07512} = 17.139. \quad (13)$$

The long distance part is given by [10]

$$(A_{RF})_L = \left(\frac{\alpha_s(1\text{GeV})}{\alpha_s(m_b)}\right)^{6/25} \left(\frac{\alpha_s(m_b)}{\alpha_s(M_Z)}\right)^{6/23} = 1.4, \quad (14)$$

from which we get

$$A_{RF} = 159.5. \quad (15)$$

For the case that the final state includes μ^+ , the strongest experimental bound for the partial decay width of proton is given by $p \rightarrow \pi^0 + \mu^+$. For simplicity, we assume $z_1 = z_2 = z_3 = 1$ for the mass eigenstates u_a^c and tune y_{bc} to normalize four-Fermi interaction as follows

$$\mathcal{L}_{eff} = \left(\frac{V}{M_P}\right)^4 \frac{A_{RF}}{M_G^2} \bar{u} \bar{d} u^c \mu^c. \quad (16)$$

From this Lagrangian, the proton decay width is given by [11]

$$\Gamma(p \rightarrow \pi^0 + \mu^+) = \frac{m_p}{64\pi f_\pi^2} \left[\left(\frac{V}{M_P}\right)^4 \frac{A_{RF}}{M_G^2} \right]^2 (1 + F + D)^2 \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right)^2 \alpha_p^2, \quad (17)$$

where F and D are chiral Lagrangian parameters, α_p is hadronic matrix element, f_π is pion decay constant and m_{π^0} and m_p are masses of pion and proton. If we put

$$F = 0.47, \quad D = 0.80, \quad \alpha_p = -0.012 \text{ GeV}^3, \quad f_\pi = 130 \text{ MeV}, \quad m_{\pi^0} = 135 \text{ MeV}, \quad m_p = 940 \text{ MeV} [12], \quad (18)$$

then we get

$$\Gamma(p \rightarrow \pi^0 + \mu^+) = (5.01 \times 10^{-12} \text{ GeV}) \left[\left(\frac{V}{M_P}\right)^4 \frac{(1000 \text{ GeV})^2}{M_G^2} \right]^2. \quad (19)$$

From the experimental bound $\tau(p \rightarrow \pi^0 + \mu^+) > 473 \times 10^{30} [\text{years}]$ [13], the upper bound for VEV is estimated as follows

$$\left[\left(\frac{V}{M_P}\right)^4 \left(\frac{1000 \text{ GeV}}{M_G}\right)^2 \right]^2 < 8.78 \times 10^{-54}. \quad (20)$$

Next, we estimate the life time of g-quark under the assumption that g-quark is lighter than scalar g-quark. For simplicity, we assume the g-quark can decay only into smuon but not into stau or squarks. With this assumption, the g-quarks decay through the following interaction

$$\mathcal{L} = \frac{(A_{RF}^z)_S V^2}{3M_P^2} (u^c + c^c + t^c) \tilde{\mu}^c (g_1 + g_2 + g_3), \quad (21)$$

from which one can see that g-quarks have the same life time. Requiring the life time of g_a is shorter than 0.1 sec as follows

$$\Gamma(g_a) = 3 \left(\frac{(A_{RF}^z)_S V^2}{3M_P^2} \right)^2 \frac{M_g}{16\pi} > \frac{1}{0.1 \text{ sec}}, \quad (22)$$

we get

$$\frac{M_g}{1000\text{GeV}} \left(\frac{V}{M_P} \right)^4 > 2.25 \times 10^{-26}, \quad (23)$$

where M_g is g-quark mass. Hereafter we assume the approximation $M_g = M_G$ is held for simplicity. From Eq.(20) and Eq.(23), the allowed region for V is given by (see Fig.1)

$$2.25 \times 10^{-26} \left(\frac{1000\text{GeV}}{M_G} \right) < \left(\frac{V}{M_P} \right)^4 < 2.96 \times 10^{-27} \left(\frac{M_G}{1000 \text{ GeV}} \right)^2. \quad (24)$$

This inequality holds when the mass bound,

$$M_G > 1.96 \text{ TeV}, \quad (25)$$

is satisfied. For example, if we put $M_G = 10 \text{ TeV}$, allowed region for V is given by

$$0.53 < \frac{V}{10^{12} \text{ GeV}} < 1.79. \quad (26)$$

Note that the factor of this constraint should not be taken seriously, because there is large model dependence.

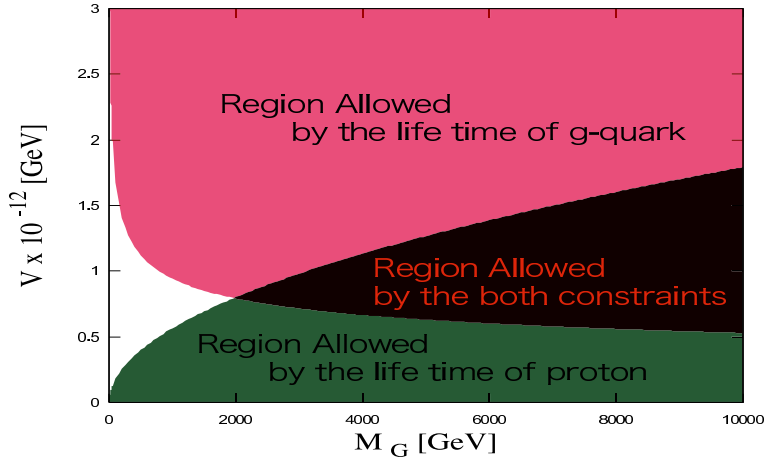


Figure 1: M_G versus V : The pink region comes from the constraint of the life time of the g-quark, which should be less than 0.1 sec. The green region comes from the constraint of the proton stability. The black region is allowed by the both constraints. The heavier of M_G , the wider the allowed region is.

4 The Maki-Nakagawa-Sakata matrix

In this section we confirm the assignment of Table 2 realizes the neutrino masses and MNS matrix. In the superpotential

$$\begin{aligned} W_L = & Y_2^N [H_1^U (L_1 N_2^c + L_2 N_1^c) + H_2^U (L_1 N_1^c - L_2 N_2^c)] \\ & + Y_3^N H_3^U L_3 N_3^c + Y_4^N L_3 (H_1^U N_1^c + H_2^U N_2^c) \\ & + Y_1^E E_1^c (H_1^D L_1 + H_2^D L_2) + Y_2^E E_2^c H_3^D L_3 + Y_3^E E_3^c (H_1^D L_2 - H_2^D L_1) \\ & + \frac{1}{2} Y_1^M \Phi_3 (N_1^c N_1^c + N_2^c N_2^c) + \frac{1}{2} Y_3^M \Phi_3 N_3^c N_3^c \\ & + \frac{1}{2} Y_2^M [2\Phi_1 N_1^c N_2^c + \Phi_2 (N_1^c N_1^c - N_2^c N_2^c)], \end{aligned} \quad (27)$$

we define the VEVs of scalar fields as follows

$$\begin{aligned}\langle H_1^U \rangle = \langle H_2^U \rangle &= \frac{1}{\sqrt{2}} v_u, & \langle H_3^U \rangle &= v'_u, & \langle H_1^D \rangle = \langle H_2^D \rangle &= \frac{1}{\sqrt{2}} v_d, & \langle H_3^D \rangle &= v'_d, \\ \langle \Phi_1 \rangle &= |a| V_0 c_N, & \langle \Phi_2 \rangle &= |a| V_0 s_N, & \langle \Phi_3 \rangle &= V_0 = \frac{V}{\sqrt{1+|a|^2}},\end{aligned}\quad (28)$$

and define the mass parameters as follows [14]

$$\begin{aligned}M_1 &= Y_1^M V_0, & M_3 &= Y_3^M V_0, & M_2 &= Y_2^M |a| V_0 \\ m_2^\nu &= Y_2^N v_u, & m_3^\nu &= |Y_3^N| v'_u, & m_4^\nu &= Y_4^N v_u, \\ m_1^l &= Y_1^E v_d, & m_2^l &= Y_2^E v'_d, & m_3^l &= Y_3^E v_d.\end{aligned}\quad (29)$$

Without loss of generality, by the field redefinition, we can define $Y_{1,2,3}^E, Y_{1,3}^M, Y_{2,4}^N$ are real and non-negative and Y_2^M, Y_3^N are complex. For simplicity, we put

$$Y_1^M = Y_3^M = 1, \quad Y_2^M = e^{i\psi}, \quad (30)$$

and

$$a = e^{i\psi} |a|, \quad M_1 = M_3 = V_0, \quad M_2 = a V_0. \quad (31)$$

With these parameters, the mass matrices are given by

$$\begin{aligned}M_l &= \frac{1}{\sqrt{2}} \begin{pmatrix} m_1^l & 0 & -m_3^l \\ m_1^l & 0 & m_3^l \\ 0 & \sqrt{2} m_2^l & 0 \end{pmatrix}, & M_D &= \frac{1}{\sqrt{2}} \begin{pmatrix} m_2^\nu & m_2^\nu & 0 \\ m_2^\nu & -m_2^\nu & 0 \\ m_4^\nu & m_4^\nu & \sqrt{2} e^{i\delta} m_3^\nu \end{pmatrix}, \\ M_R &= V_0 \begin{pmatrix} 1 + a s_N & a c_N & 0 \\ a c_N & 1 - a s_N & 0 \\ 0 & 0 & 1 \end{pmatrix}.\end{aligned}\quad (32)$$

Due to the seesaw mechanism, the neutrino mass matrix is given by

$$\begin{aligned}M_\nu &= M_D M_R^{-1} M_D^t = \frac{1}{1-a^2} \begin{pmatrix} \rho_2^2(1-ac_N) & -\rho_2^2 a s_N & \rho_2 \rho_4(1-ac_N) \\ -\rho_2^2 a s_N & \rho_2^2(1+ac_N) & -\rho_2 \rho_4 a s_N \\ \rho_2 \rho_4(1-ac_N) & -\rho_2 \rho_4 a s_N & \rho_4^2(1-ac_N) + \rho_3^2(1-a^2) \end{pmatrix}, \\ \rho_2 &= \frac{m_2^\nu}{\sqrt{V_0}}, \quad \rho_4 = \frac{m_4^\nu}{\sqrt{V_0}}, \quad \rho_3 = \frac{e^{i\delta} m_3^\nu}{\sqrt{V_0}}.\end{aligned}\quad (33)$$

The charged lepton mass matrix is diagonalized as follows

$$V_l^\dagger M_l^* M_l^t V_l = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) = ((m_2^l)^2, (m_3^l)^2, (m_1^l)^2), \quad (34)$$

$$V_l = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ -\sqrt{2} & 0 & 0 \end{pmatrix}. \quad (35)$$

To realize experimental results [15], the neutrino mass matrix should be diagonalized as follows

$$\begin{aligned}V_\nu^\dagger M_\nu V_\nu &= \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \\ V_{MNS} &= V_l^\dagger V_\nu = \begin{pmatrix} -c_\nu c_{13} & -s_\nu c_{13} & s_{13} \\ \frac{1}{\sqrt{2}} s_\nu + \frac{1}{\sqrt{2}} c_\nu s_{13} & -\frac{1}{\sqrt{2}} c_\nu + \frac{1}{\sqrt{2}} s_\nu s_{13} & \frac{1}{\sqrt{2}} c_{13} \\ -\frac{1}{\sqrt{2}} s_\nu + \frac{1}{\sqrt{2}} c_\nu s_{13} & \frac{1}{\sqrt{2}} c_\nu + \frac{1}{\sqrt{2}} s_\nu s_{13} & \frac{1}{\sqrt{2}} c_{13} \end{pmatrix}, \\ \sin^2 2\theta_\nu &= 0.8704, \quad \sin^2 2\theta_{13} = 0.11 \\ m_{\nu_2}^2 - m_{\nu_1}^2 &= 7.6 \times 10^{-5} [\text{eV}^2], \quad m_{\nu_3}^2 - m_{\nu_2}^2 = 2.4 \times 10^{-3} [\text{eV}^2].\end{aligned}\quad (36)$$

Due to the overabundance of parameters, unfortunately, it is impossible to fix the parameters by these constraints. Therefore we assume a and ρ_3^2 are real for simplicity, then we get

$$\begin{aligned}a &= 1.40321, \quad s_N = 0.110194, \\ \rho_2^2 &= 0.0212781 [\text{eV}], \quad \rho_3^2 = -0.0539146 [\text{eV}], \quad \rho_4^2 = 0.112142 [\text{eV}], \\ m_{\nu_1} &= -0.0207554 [\text{eV}], \quad m_{\nu_2} = 0.0225119 [\text{eV}], \quad m_{\nu_3} = -0.0539146 [\text{eV}],\end{aligned}\quad (37)$$

and

$$V = \sqrt{1 + |a|^2} \frac{(Y_4^N v_u)^2}{\rho_4^2} = (1.54 \times 10^{12}) \left(Y_4^N \frac{v_u}{10 \text{ GeV}} \right)^2 [\text{GeV}]. \quad (38)$$

From the requirement of perturbativity of Yukawa coupling such as $Y_4^N < 1$, V is bounded from above. It is difficult for V to be much larger than $O(10^{12} \text{ GeV})$. Therefore, there exists upper bound for V even in the case that proton decay is perfectly forbidden. It must be noted that there is non-trivial coincidence of the constraint of Eq.(26) with RHN mass scale.

Finally we give a comment about how to realize the VEV directions given in Eq.(28). The Higgs potential derived from superpotential

$$\begin{aligned} W_H &= \lambda_1 S_3 (H_1^U H_1^D + H_2^U H_2^D) + \lambda_3 S_3 H_3^U H_3^D \\ &+ \lambda_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 (S_1 H_1^U + S_2 H_2^U) H_3^D, \end{aligned} \quad (39)$$

has accidental $O(2)$ symmetry. To avoid massless Nambu-Goldstone boson, we add soft $S_4 \times Z_2$ breaking terms in the form of the inner products with $(1, 1)$ as follows

$$\mathcal{L} \supset m_{BU}^2 (H_3^U)^\dagger (H_1^U + H_2^U) + m_{BD}^2 (H_3^D)^\dagger (H_1^D + H_2^D) + m_{BS}^2 (S_3)^\dagger (S_1 + S_2) + h.c., \quad (40)$$

then the VEV direction $(A_1, A_2) \propto (1, 1)$ ($A = S, H^U, H^D$) becomes the minimum of potential and the VEV direction of H_a^U, H_a^D, S_a in Eq.(28) is realized. The potential of Φ_a, Φ_a^c derived from the leading order superpotential

$$\begin{aligned} W_\Phi &= \frac{A_1}{2M_P} \Phi_3^2 [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\ &+ \frac{A_2}{2M_P} (\Phi_1^2 + \Phi_2^2) [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\ &+ \frac{A_3}{2M_P} \left\{ 2\sqrt{3}\Phi_1\Phi_2 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + (\Phi_1^2 - \Phi_2^2) [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\} \\ &+ \frac{A_4}{2M_P} \Phi_3 \left\{ \sqrt{3}\Phi_1 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + \Phi_2 [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\}, \end{aligned} \quad (41)$$

does not have accidental symmetry. To avoid domain wall problem, we must add soft S_4 breaking terms. Then the VEV direction of Φ, Φ^c is controlled by the parameters $A_{1,2,3,4}$ and soft SUSY and flavor breaking parameters. The mechanism for inducing soft flavor symmetry breaking terms is unknown and beyond the scope of this paper. We leave it for future work.

5 The origin of the scale of V

Finally we explain how the required value for V is realized. The superpotential Eq.(41) is simplified as follows

$$W_\Phi = \frac{A}{M_P} (\Phi \Phi^c)^2. \quad (42)$$

The origin of Φ -potential becomes unstable point due to the negative soft SUSY breaking squared mass and the potential is lifted by F-term derived by W_Φ . Minimizing the potential, the VEV of Φ is estimated as follows

$$V \sim \langle \Phi \rangle = \left(\frac{m_{\text{SUSY}} M_P}{A} \right)^{\frac{1}{2}}. \quad (43)$$

For the typical range of A and SUSY breaking scale m_{SUSY} such as $0.01 < A < 1$, $0.1 \text{ TeV} < m_{\text{SUSY}} < 10 \text{ TeV}$. Hence the region of V is as follows

$$10^{10} \text{ GeV} < V < 10^{12} \text{ GeV}. \quad (44)$$

Although the each of region given in Eq.(26) and Eq.(44) is very narrow, remarkably, there exists overlap.

6 Conclusion

We have considered the suppression mechanism of proton decay based on $S_4 \times Z_2$ flavor symmetric model. Under the field assignment that MNS matrix is realized, we have classified the several suppression mechanisms. There are two new solutions other than the well known leptoquark solution. For the cancellation solution, the four-Fermi interaction which induces proton decay vanishes in a special VEV direction. For the suppression solution, the stability of proton is satisfied for appropriate size of VEV. Although the allowed region for the VEV is very narrow, there is coincidence between the allowed regions required by the different phenomenological considerations such as naive potential analysis, RHN mass scale and the life times of g-quark and proton.

A Mixing matrix of scalar g-quarks

Here we define the mixing matrix of scalar g-quarks. The mass terms of scalar g-quarks are given as follows

$$\begin{aligned}
-\mathcal{L} &\supset m_G^2(|G_1|^2 + |G_2|^2 + |G_3|^2) + m_{G^c}^2(|G_1^c|^2 + |G_2^c|^2 + |G_3^c|^2) \\
&+ kA_k[S_3(G_1G_1^c + G_2G_2^c + G_3G_3^c) + h.c.] \\
&+ |k(G_1G_1^c + G_2G_2^c + G_3G_3^c) + \lambda_1(H_1^U H_1^D + H_2^U H_2^D) + \lambda_3 H_3^U H_3^D|^2 \\
&+ |k|^2|S_3|^2(|G_1|^2 + |G_2|^2 + |G_3|^2 + |G_1^c|^2 + |G_2^c|^2 + |G_3^c|^2) + \text{D-terms} \\
&= \sum_a (G_a^*, G_a^c) \begin{pmatrix} M_{++}^2 & M_{+-}^2 \\ M_{-+}^2 & M_{--}^2 \end{pmatrix} \begin{pmatrix} G_a \\ (G_a^c)^* \end{pmatrix}, \tag{45}
\end{aligned}$$

where we assumed A_k is real for simplicity. If

$$M_{+-}^2 = kA_k v'_s + k(\lambda_1 v_u v_d + \lambda_3 v'_u v'_d) = 0, \tag{46}$$

is satisfied, then there is no $G - G^c$ mixing. In the case that $M_{+-}^2 \neq 0$, the mixing matrix of scalar g-quarks is defined as follows

$$V_G = \begin{pmatrix} c_G & -s_G \\ s_G & c_G \end{pmatrix}, \quad V_G^T \begin{pmatrix} M_{++}^2 & M_{+-}^2 \\ M_{-+}^2 & M_{--}^2 \end{pmatrix} V_G = \text{diag}(M_+^2, M_-^2), \quad \begin{pmatrix} G_a \\ (G_a^c)^* \end{pmatrix} = V_G \begin{pmatrix} G_{+,a} \\ G_{-,a} \end{pmatrix}. \tag{47}$$

From this definition, the propagators of scalar g-quarks are given by

$$\begin{aligned}
\langle G_a, G_b^* \rangle &= \delta_{ab} \left(\frac{c_G^2}{M_+^2} + \frac{s_G^2}{M_-^2} \right) = \delta_{ab} \frac{C_{GG}}{M_G^2}, \quad C_{GG} = c_G^2 \frac{M_+^2}{M_+^2} + s_G^2 \frac{M_-^2}{M_-^2}, \\
\langle G_a, G_b^c \rangle &= \delta_{ab} c_G s_G \left(\frac{1}{M_+^2} - \frac{1}{M_-^2} \right) = \delta_{ab} \frac{C_{GG^c}}{M_G^2}, \quad C_{GG^c} = c_G s_G \left(\frac{M_+^2}{M_+^2} - \frac{M_-^2}{M_-^2} \right), \tag{48}
\end{aligned}$$

where $M_G = \sqrt{M_+ M_-}$ is mean scalar g-quark mass.

References

- [1] H. P. Nilles, Phys. Rep. **110** (1984) 1.
- [2] D. Suematsu and Y. Yamagishi, Int. J. Mod. Phys. **A10** (1995) 4521.
- [3] Y. Daikoku and H. Okada, Phys. Rev. **D82** (2010) 033007[arXiv:0910.3370[hep-ph]].
- [4] F. Zwirner, Int. J. Mod. Phys. **A3** (1988) 49, J. L. Hewett and T. G. Rizzo, Phys. Rep. **183** (1989) 193.
- [5] B. A. Campbell, J. Ellis, K. Enqvist, M. K. Gaillard and D. V. Nanopoulos, Int. J. Mod. Phys. **A2** (1987) 831; Y. Daikoku and H. Okada, [arXiv:1008.0914 [hep-ph]].
- [6] Y. Daikoku, H. Okada and T. Toma, Prog. Theor. Phys. **126** (2011) 855-883 [arXiv:1106.4717 hep-ph].
- [7] M. Kawasaki, K. Kohri and T. Moroi, Phys. Rev. **D71** (2005) 083502 [astro-ph/0408426].

- [8] R. Howl and S. F. King, JHEP**0805** (2008) 008[arXiv:0802.1909[hep-ph]].
- [9] J. Hisano, [hep-ph/0004266].
- [10] P. Nath, and P. F. Perez, Phys. Rep. **441** (2007) 191 [hep-ph/0601023].
- [11] T. Goto and T. Nihei, Phys. Rev. **D59** (1999) 115009[hep-ph/9808255].
- [12] Y. Aoki, C. Dawson, J. Noaki, and A. Soni, Phys. Rev. **D75** (2007) 014507[hep-lat/0607002].
- [13] Particle Data Group, J. Phys. **G37** (2010) 075021 and 2011 partial update for the 2012 edition .
- [14] J.Kubo, Phys. Lett. **B578** (2004) 156.
- [15] T2K Collaboration: K. Abe et.al, Phys. Rev. Lett. **107** (2011) 041801[arXiv:11062822[hep-ex]].